Discounting – Other topics

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Replicating portfolios

Replicating portfolios – Definition

General background

- A **replicating portfolio** is a portfolio of **assets** that replicates the **cash flow** profile of **liabilities** as closely as possible under a given set of economic conditions
- Typically, assets in the portfolio have **simple pricing formulae**, enabling quick pricing in a large number of scenarios.
- Used as a tool for speeding up stochastic runs and avoiding **nested simulations**
- Note that the replicating-portfolio approach is an **approximation**. Complex guarantee liabilities cannot as a rule be valued exactly using simple formulae
- For IFRS 17 purposes, these are not seen as approximations but really used as an exact replacement.

IFRS 17 definition

- *A **replicating portfolio** is a theoretical portfolio of assets providing cash flows that exactly match the cash flows from the liability in all scenarios.*

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Replicating portfolios – IFRS 17 Relevance

Replicating portfolios are referenced in the market variables section of the guidance

...If a replicating portfolio of assets exists for some of the cash flows that arise from a group of insurance contracts, the entity can use the fair value of those assets to measure the relevant fulfilment cash flows instead of explicitly estimating the cash flows and discount rate...

...IFRS 17 does not require an entity to use a replicating portfolio technique. However,... the entity shall satisfy itself that a replicating portfolio technique would be unlikely to lead to a materially different measurement...

Challenges of replicating portfolios are also acknowledged in IFRS 17

...using a replicating portfolio technique might require splitting the cash flows of the insurance contracts...

...many stakeholders argued that it is impossible to split the cash flows in this way...

...IFRS 17 permits, but does not require, the use of a replicating portfolio technique and allows other approaches....

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Replicating portfolios – Theory

Market value of a liability portfolio is defined as follows:

$$MV\ Value_{liab} = \mathrm{E}_{\mathrm{RN}}\left[\sum_{\mathrm{t=1}}^{\mathrm{T}} \mathrm{CF(t)}_{liab} \times D_t\right] = \frac{1}{\mathrm{n}} \sum_{\mathrm{t=1}}^{\mathrm{T}} \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{CF(i,t)}_{\mathrm{liab}} \times D(i,t)$$

Where **T** is the time horizon and **n** is the number of stochastic scenarios.

Market value of the asset portfolio is defined as:

$$MV\ Value_{RP} = \frac{1}{n} \beta_t \sum_{p=1}^{P} \sum_{t=1}^{T} \sum_{i=1}^{n} CF_p(i,t) \times D(i,t)$$

Where **P** is the number of replicating instruments, $D(i,t)$ is the stochastic discount factor at time t for simulation i, and β represents the weights of assets in the replicating portfolio

Replicating portfolios – Theory

Typical optimisation targets include:

Cash flows

$$\min_{\beta} \sum_{n} \sum_{t} \left\| \mathrm{CFL}_{n,t} - \beta \sum_{p} \mathrm{CFA}^{p}_{n,t} \right\|$$

Market values

$$\min_{\beta} \sum_{n} \left\| MVL_n - \beta \sum_{p} MVA_{n,p} \right\|$$

MV & Sensitivities

$$\min_{\beta} \sum_{n} \left\{ \left\| MVL_n - \beta \sum_{p} MVA_{n,p} \right\| + \left\| \Delta L_n - \beta \sum_{p} \Delta A_{n,p} \right\| \right\}$$

Optimisation based on cash flows may be impractical due high dimensionality. It is often more practical to match **terminal or present value cash flows**.

Replicating portfolios – Theory

Instruments

Possible instrument choices are:

- Zero Coupon Bonds
- Forward Swap
- Receiver/Payer/Straddle Swaption
- Constant Maturity Swap
- Rate Floors/Caps
- Equity Index
- Equity Index Put/Call/Loopback Option/Straddle
- FX Forward/Put/Call

Typical problems

- Over-fitting / Bar-belling
- Many similar assets / Co-linearity
- Availability of market prices for long term durations (liquidity)
- Trading costs (bid-ask spreads)
- Explaining / making sense of results
- Only work for financial risks

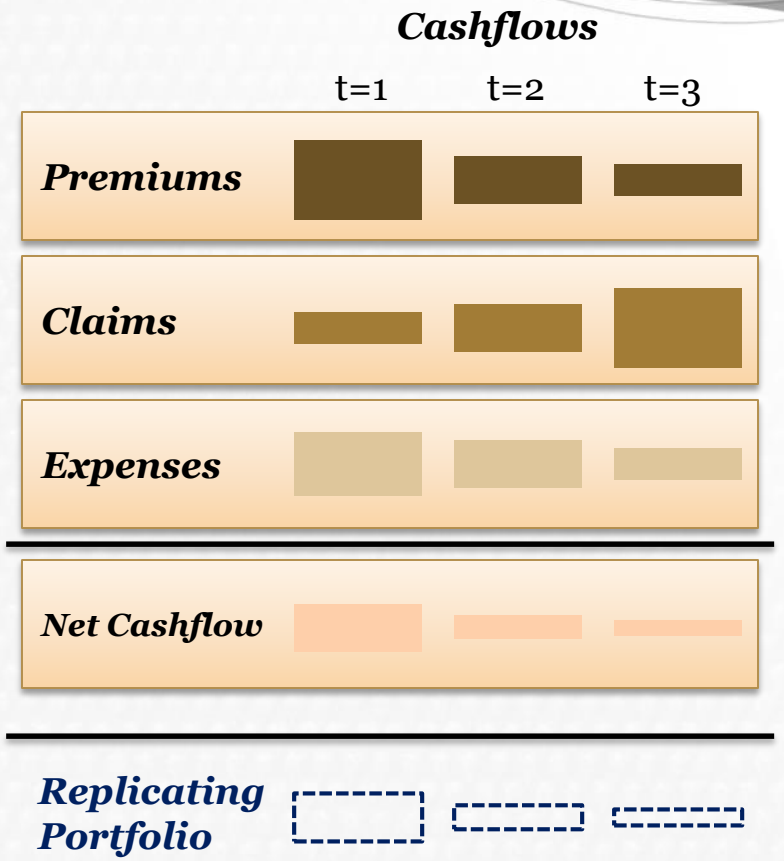
RP issues specific to insurance portfolios

- Most products have simultaneous dependence on market and non-market risks
- Non-market risks such as mortality/longevity and some policyholder behavior cannot be replicated, even with exotic financial instruments
- Difficult or impossible to separate the market and non-market risks and cash flows
- Need to find a balance between the benefits of a closed-form replicating portfolio and one requiring more sophisticated, but time consuming valuation methods

Replicating portfolios – Example

- A portfolio of assets is constructed which replicates the liability cash flows profile

- The market-consistent value of liabilities can then be determined by calculating the value of the proxy asset portfolio



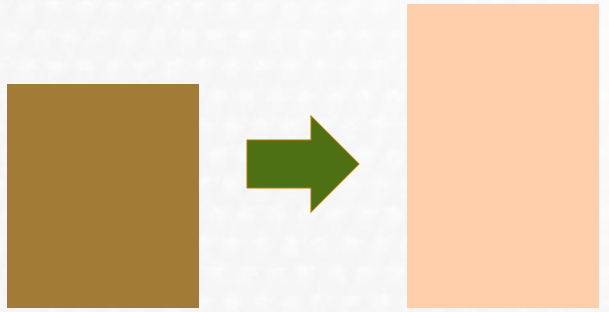
Replicating portfolios – Example

- Insurer with 3 insurance products

- Product characteristics:

1. Initial investment by policyholder
2. Fund value is paid to policyholder after 1 year
3. 3 types of products:
   a) 100 invested in equities
   b) 100 invested in equities, guaranteed to return at least the initial investment
   c) 100 invested in equities, guaranteed to return at least 10%

- **Projection of Liabilities**



Initial Investment

Value after 1 year

Replicating portfolios – Example

- Simplified Example:

- 3 assets available to replicate:
- Equity Shares
- Zero Coupon Bond
- Call Option with Strike = 100

- Objective: minimize difference between ending cashflows across defined scenario set

Value of Investment After 1 Year

Scenario	Equity Return	Product a	Product b	Product c
1	-14,4%	85,6	100,0	110,0
2	-9,1%	90,9	100,0	110,0
3	-5,9%	94,1	100,0	110,0
4	-0,2%	99,8	100,0	110,0
5	1,6%	101,6	101,6	110,0
6	3,3%	103,3	103,3	110,0
7	5,7%	105,7	105,7	110,0
8	7,2%	107,2	107,2	110,0
9	13,7%	113,7	113,7	113,7
10	18,2%	118,2	118,2	118,2

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Replicating portfolios – Example

Table 1 – The Replicating Portfolio of Assets

Number of Assets to Purchase	Product a (no guarantee)	Product b (0% guarantee)	Product c (10% guarantee)
Zero Coupon Bond		100	119
Equity Shares	100		-11
Call Option (X=100)		100	56

% of Portfolio Value	Product a (no guarantee)	Product b (0% guarantee)	Product c (10% guarantee)
Zero Coupon Bond		90%	105%
Equity Shares	100%		-10%
Call Option (X=100)		10%	5%

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Replicating portfolios – Example

Table 2 – How do Replicating Portfolios help?

	Product a (no guarantee)	Product b (0% guarantee)	Product c (10% guarantee)
Market Value	**100.0**	**105.6**	**110.7**
Replicating Portfolio	100.0	105.6	108.5

- Product a and b are replicated perfectly
- Product c has some slippage due to lack of available assets that perfectly replicate liability cashflows

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Stochastic scenarios

Stochastic scenarios – IFRS 17 references

IFRS 17 guidance on projecting cash flows introduces the idea that scenarios are needed

> *...an entity shall estimate the expected value (ie the probability-weighted mean) of the full range of possible outcomes...*

> **This typically implies the mean of multiple scenarios**

The standard further explains when stochastic scenarios would be needed

> *...in some cases, the cash flows may be driven by complex underlying factors and may respond in a non-linear fashion to changes in economic conditions... In such cases, more sophisticated stochastic modelling is likely to be necessary to satisfy the measurement objective...*

Stochastic scenarios – IFRS 17 references

Observable inputs should recreate observable prices

> *...Judgement is required to determine the technique that best meets the objective of consistency with observable market variables in specific circumstances. In particular, the technique used must result in the measurement of any options and guarantees included in the insurance contracts being consistent with observable market prices (if any) for such options and guarantees...*

Stochastic economic scenarios are market variables. These must maximize/focus on market and not entity specific assumptions

> *... An entity shall maximise the use of observable inputs and shall not substitute its own estimates for observable market data except as described in paragraph 79 of IFRS 13 Fair Value Measurement...*

The most common methods for measuring financial options and guarantees on a market consistent, stochastic basis are the 'risk neutral' and 'real world/deflator' methods. In some cases, closed form solutions, such as Black Scholes may be used.

Stochastic scenarios – Risk neutral

Risk neutral methods:

- Financial options and guarantees measured consistently with the cost of hedging the obligation (where observable) at the balance sheet date.
- Achieved through the modelling of the interactions between cash flows that vary with underlying items and the discount rate for the contract as a whole
- When modeled, the interaction between rates could include policyholder behavior changes. However, these types of assumptions cannot typically be observed so management judgment is used.

Alternative 'real world' stochastic methods where some asset classes are assumed to earn a credit spread, based on historical market averages or management judgement are not permitted under IFRS 17

- Financial options and guarantees would not then be measured consistently with observable current market prices.

Stochastic scenarios – Unobservable inputs

In developing market assumptions, several will not be observable. Examples include:

- Fixed income rates beyond 30 years (or much shorter in some territories)
- Long term volatility
- Correlations across multiple market assumptions
- Any others, by territory, where a market does not exist

While discussing yield curve estimation, IFRS 17 addresses unobservable inputs

> *...develop unobservable inputs using the best information available in the circumstances. Such inputs might include the entity's own data and, in the context of IFRS 17, the entity might place more weight on long-term estimates than on short-term fluctuations...*
>
> *...adjust those data to reflect all information about market participant assumptions that is reasonably available...*

Stochastic scenarios – Economic scenario generators

Economic scenario generators are used to simulate future economic events and financial markets

Models developed for fixed income and equity projections

- Volatility
- Mean reversion
- Equity jumps
- Regime switching

Risk neutral scenarios must satisfy no arbitrage / martingale tests

- Cannot generate riskless profits
- Reproduce derivative prices

For some processes, variance reduction techniques may be helpful to reduce run time or increase precision

Growth and inflation rates

Growth and inflation rates – IFRS 17 guidance

Companies may reflect inflation in rates or cash flows

...nominal cash flows (ie those that include the effect of inflation) shall be discounted at rates that include the effect of inflation...

...real cash flows (ie those that exclude the effect of inflation) shall be discounted at rates that exclude the effect of inflation...

Inclusion of inflation should be consistent with underlying assumptions

...Estimated probabilities for non-market variables shall not contradict observable market variables. For example, estimated probabilities for future inflation rate scenarios shall be as consistent as possible with probabilities implied by market interest rates...

There are presentation subtleties with respect to inflation

...assumptions about inflation based on an index of prices or rates or on prices of assets with inflation-linked returns are financial assumptions...

...assumptions about inflation based on an entity's expectation of specific price changes are non-financial assumptions...

Cash flows that vary based on financial underlying

Similar to nominal versus real cash flows, insurers may adjust for these

...cash flows that vary based on the returns on any financial underlying items shall be:

(i) discounted using rates that reflect that variability; or

(ii) adjusted for the effect of that variability and discounted at a rate

Regardless of approach used, the result should be the same. However, when guarantees are in place, the adjustment may be a challenge.

For example, consider a universal life type product with a 3% guarantee. The liability rate is 4%, the asset rate (net of expected defaults) is 5% and management intends to share 50% of any excess return with the policyholder. Alternative approaches might be:

- Project cash flows and discount at 4.5% - reflects variability
- Project cash flows and discount at something between 4% and 4.5% - notes that only a portion of cash flows vary
- Project cash flows and discount at 4% - remove variability and credit liability rate
- Project cash flows at 3% and discount at 4% - remove variability and credit guarantee rate

Which is appropriate?

Cash flows that vary based on financial underlying
Reflecting variability

Further guidance on reflecting the variability indicate that it should be reflected regardless of whether:

- the variability is due to contractual terms or is due to discretion
- the entity holds the underlying or not

This may imply that entities would make this adjustment both contracts that meet the variable fee requirements and those that do not provided they reflect this variability

Entities are not required to split the cash flows that do or do not vary based on underlying. If they do split these cash flows, they must apply discount rates that are appropriate for the cash flows as a whole

- Possible methods suggested include stochastic modeling or risk neutral measurement techniques
- Measurement of any option or guarantee included in the insurance contract must be consistent with observable market prices

This may imply that the discount rate should remove the impact of any credit (or other) risk impact that is not consistent with market prices

Cash flows that vary based on financial underlying
Reflecting variability

Slightly different / expanded example:

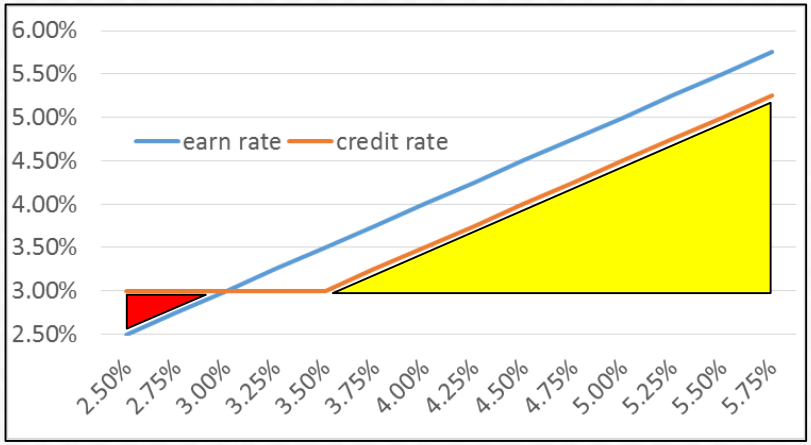
- Rates in the box, assume a 10 year product with CU 100 deposit

Need to reflect the option and guarantee but what if one could earn a riskless excess yield?

- Removing variability would result in crediting 3% and discounting at 3.5%. Initial liability (before CSM) would be CU 100 x (1.03/1.035)^10 = CU 95.27
- If variability was included, then value should remain the same. This implies that the discount rate would be 4.254%: CU 100 x (1.0375/1.04254)^10 = CU 95.27

But no one can earn a riskless excess yield and this ignores the crediting rate asymmetry. Compare to a product that credits 3% (yellow triangle) regardless of asset yield. Or a product with no guarantee (red triangle).

Guaranteed crediting rate	3.00%
Liability rate	3.50%
Implicit liquidity premium	0.50%
Expected underlying rate	4.25%
Discretion	Earn less 50bp
Expected crediting rate	3.75%



Cash flows that vary based on financial underlying
Reflecting variability

One possible approach may be to run a stochastic approach around the liability rate but also reflects the 3% guarantee:

- Assume the market believes there are seven equally likely potential scenarios: 2.75% to 4.25% increasing by 25bps each (mean is 3.5%)
- Value = 100 x (1+disc rate)/(1+credit rate)^10 = 97.32
- This appears to reflect the variability of the cash flows and the guarantee while not introducing any spread beyond the liquidity premium in the liability rate.
- The spread in the expected yield over the liability discount rate would be recognized as it was realized
- An alternate method may be to increase both the mean of the crediting and discount rate

Discount rate	Crediting rate	Value
2.75%	3.00%	102.46
3.00%	3.00%	100.00
3.25%	3.00%	97.60
3.50%	3.00%	95.27
3.75%	3.25%	95.28
4.00%	3.50%	95.29
4.25%	3.75%	95.31
3.50%	**Mean**	**97.32**

Regardless of method chosen, entities should ensure the results are consistent with market prices or market participant assumptions

Discounting – Other topics

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